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Minkowski Brane in Asymptotic dS_5 Spacetime without Fine-tuning

Zhe Chang[1], Shao-Xia Chen[2] and Xin-Bing Huang[3]

Institute of High Energy Physics

Chinese Academy of Sciences

P.O.Box 918(4), 100039 Beijing, China

Abstract

We discuss properties of a 3-brane in an asymptotic 5-dimensional de-Sitter spacetime. It is found that a Minkowski solution can be obtained without fine-tuning. In the model, the tiny observed positive cosmological constant is interpreted as a curvature of 5-dimensional manifold, but the Minkowski spacetime, where we live, is a natural 3-brane perpendicular to the fifth coordinate axis.

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The tiny but not zero cosmological constant has puzzled astrophysicists and physicists for nearly a century[1]. Various attempts have been made in trying to solve the puzzle. However, up to now, there hasn't been a theory that can give a cosmological constant whose order is the same as that of the observed value.

New progress made in this direction came from the brane world picture of cosmology, and in particular, the Randall-Sundrum (RS) scenario[2, 3]. The RS model

[1]changz@mail.ihep.ac.cn
[2]ruxanna@mail.ihep.ac.cn
[3]huangxb@mail.ihep.ac.cn

was proposed by Randall and Sundrum to aim at a reasonable explanation of the hierarchy between the electro-weak scale and Planck scale in the four-dimensional effective field theory. In this scenario, the spacetime is five-dimensional. We live in a 3-brane, which is perpendicular to the fifth coordinate axis. The fifth dimension can be compact or noncompact. All matter and interactions except gravity are confined to the 3-brane. Since the RS model was proposed, many physicists[4]-[18] have been trying to apply it to such issues as the hierarchy problem[11], cosmological constant problem[4, 8, 9, 11, 13, 14, 17, 18], localization of gravity[7], dynamics of the brane[5], and the realization of the AdS_5 spacetime in string theory.

In this Letter, we consider an asymptotic dS_5 spacetime, which is characterized by a positive cosmological constant. It is found that there is a natural solution of Minkowski brane without fine-tuning.

We start from the action of the dS 5-dimensional spacetime with a dilaton field ϕ ($\equiv \phi(x_5)$)

$$S = \int d^5x \left(\sqrt{-G} \left(R - \frac{4}{3}(\nabla\phi)^2 - \Lambda e^{a\phi} \right) - \sqrt{-g}\delta(x_5) V e^{b\phi} \right) \ , \tag{1}$$

where G and g are determinants of the five-dimensional spacetime metric G_{MN} and the four-dimensional brane metric $g_{\mu\nu}$, respectively. The Einstein equation corresponding to the action reads

$$\begin{aligned} \sqrt{-G} \left(R_{MN} - \frac{1}{2} G_{MN} R \right) - \frac{4}{3}\sqrt{-G} \left(\nabla_M\phi\nabla_N\phi - \frac{1}{2} G_{MN} (\nabla\phi)^2 \right) & \\ + \frac{1}{2} \left(\Lambda e^{a\phi} \sqrt{-G} G_{MN} + \sqrt{-g} V e^{b\phi} g_{\mu\nu} \delta^\mu_M \delta^\nu_N \delta(x_5) \right) = 0 \ . & \end{aligned} \tag{2}$$

The equation of motion of the dilaton is as following

$$\sqrt{-G} \left(\frac{8}{3}\nabla^2\phi - a\Lambda e^{a\phi} \right) - b\sqrt{-g} V \delta(x_5) e^{b\phi} = 0 \ . \tag{3}$$

Following Randall and Sundrum, we assume that the metric of the five-dimensional spacetime is of the form

$$ds^2 = e^{2A(x_5)} g_{\mu\nu} dx^\mu dx^\nu + (dx^5)^2 \ , \tag{4}$$

where

$$g_{\mu\nu} = \mathrm{diag}\left(-e^{2\sqrt{-\bar{\Lambda}}x_4},\ e^{2\sqrt{-\bar{\Lambda}}x_4},\ e^{2\sqrt{-\bar{\Lambda}}x_4},\ 1 \right) \ . \tag{5}$$

Please note that, in general, the 3−brane can be any symmetric space. With the above ansatz for the metrics on 5-dimensional spacetime and 3-brane, we transform the equations(3) and (2) into the form

$$\frac{8}{3}\phi'' + \frac{32}{3}A'\phi' - a\Lambda e^{a\phi} - bV\delta(x_5)e^{b\phi} = 0 \ , \tag{6}$$

$$3A'' + \frac{4}{3}(\phi')^2 + 3\bar{\Lambda}e^{-2A} + \frac{1}{2}Ve^{b\phi}\delta(x_5) = 0 \ , \tag{7}$$

$$6(A')^2 - 6\bar{\Lambda}e^{-2A} - \frac{2}{3}(\phi')^2 + \frac{1}{2}\Lambda e^{a\phi} = 0 \ , \tag{8}$$

where the notation $'$ denotes differentiation with respect to x_5. We are interest on finding a natural Minkowski solution of the brane system without fine-tuning.

Away from the 3-brane, the equations of motion of the system (bulk equations) reduce to the form

$$\frac{8}{3}\phi'' + \frac{32}{3}A'\phi' - a\Lambda e^{a\phi} = 0 \ , \tag{9}$$

$$3A'' + \frac{4}{3}(\phi')^2 + 3\bar{\Lambda}e^{-2A} = 0 \ , \tag{10}$$

$$6(A')^2 - 6\bar{\Lambda}e^{-2A} - \frac{2}{3}(\phi')^2 + \frac{1}{2}\Lambda e^{a\phi} = 0 \ . \tag{11}$$

The difficulty in trying to solve the bulk equations comes from the $\bar{\Lambda}$ terms. Here, we present a natural Minkowskin solution of the bulk equations of motion in integration form

$$\begin{aligned} A(x_5) &= -\frac{2\alpha}{a}\ln\left(a\sqrt{D}\left[1 - 2H(x_5)\right]x_5 + 2d \right) - e^{c-[2H(x_5)-1]x_5} + hx + \eta \ , \\ \phi(x_5) &= -\left[2H(x) - 1\right]\int_{0^\pm}^{x_5} dx \sqrt{\frac{D}{\left(\frac{a\sqrt{D}[1-2H(x)]x}{2} + d\right)^2} + \frac{9}{4}e^{c-[2H(x)-1]x}} \ , \\ \bar{\Lambda} &= 0 \ , \end{aligned} \tag{12}$$

where $D \equiv \dfrac{3a\Lambda}{4(a+8\alpha)}$, $c,\ h,\ d,\ \alpha$ and η are constants, and $H(x)$ is the Heviside function

$$H(x) = \begin{cases} 1, & \text{for } \ x > 0 \ , \\ 0, & \text{for } \ x < 0 \ . \end{cases}$$

In figures 1 and 2, we give a plot of the dilaton field $\phi(x_5)$ with selected parameters $a_- = 0.189,\ d_- = 1.545,\ c_- = -28.481,\ \Lambda_- = 10^{-4},\ \alpha = 8.776$ and $h_- = -4.01 \times 10^{-3}$ for $x < 0$; $a_+ = 0.01,\ d_+ = 15.875,\ c_+ = -19.63,\ \Lambda_+ = 10^{-4},\ \alpha = 413.56$ and $h_+ = -8.306 \times 10^{-3}$ for $x > 0$.

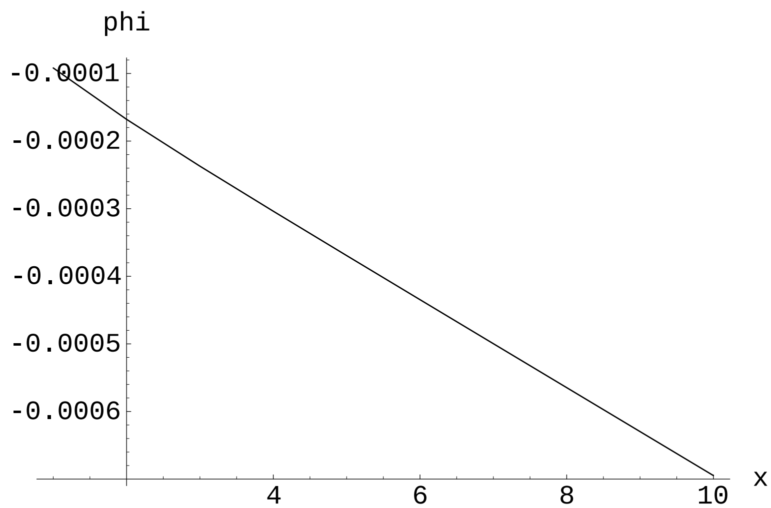

FIG.1 The behavior of the dilaton field $\phi(x)$.

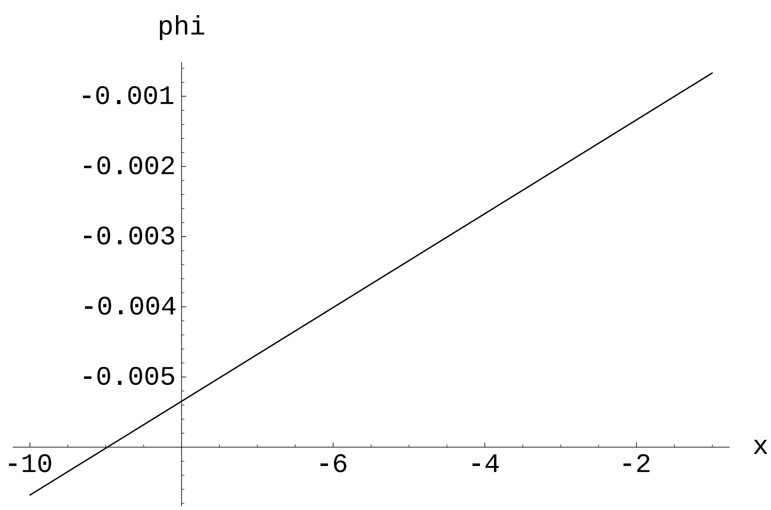

FIG.2 The behavior of the dilaton field $\phi(x)$ for $x < 0$.

The matching conditions on the brane for Minkowski solution (12) are of the form

$$\sqrt{\frac{D_+}{d_+^2}+\frac{9}{4}e^{c_+}}+\sqrt{\frac{D_-}{d_-^2}+\frac{9}{4}e^{c_-}}=-\frac{3}{8}bVe^{b\phi(0)}\ , \qquad (13)$$

$$-\frac{8\sqrt{D_+}}{9a_+d_+}+e^{c_+}+h_+-\frac{8\sqrt{D_-}}{9a_-d_-}+e^{c_-}-h_-=-\frac{1}{6}Ve^{b\phi(0)}\ . \qquad (14)$$

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